

Many-fingered time Bohmian mechanics

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Abstract

The many-fingered time (MFT) formulation of many-particle quantum mechanics and quantum field theory is a natural framework that overcomes the problem of “instantaneous collapse” in entangled systems that exhibit nonlocalities. The corresponding Bohmian interpretation can also be formulated in terms of MFT beables, which alleviates the problem of instantaneous action at a distance by using an ontology that differs from that in the standard Bohmian interpretation. The appearance of usual single-time particle-positions and fields is recovered by quantum measurements.

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1 Introduction

Entanglement in quantum mechanics (QM) induces certain nonlocal features of QM. While there is still some controversy regarding the question if orthodox QM itself is really an intrinsically nonlocal theory (see e.g. [1] and references therein), from the work of John Bell [2] it is clear that any hidden-variable interpretation of QM must be explicitly nonlocal. The best known and most successful nonlocal hidden-variable interpretation of QM and quantum field theory (QFT) is the Bohmian interpretation [3, 4, 5, 6, 7, 8]. A typical property of this interpretation is an instantaneous action at a distance among the hidden-variables – particle-positions and field-configurations. The word “instantaneous” requires a preferred global choice of the time-coordinate, which seems to contradict the principle of relativity. A possible way out of this problem is to introduce a “preferred” foliation of spacetime in a dynamical way [9, 10, 11]. Another possibility is to introduce a Bohmian equation of motion not only for space-coordinates of particles, but also for their time-coordinates [12, 13].

The most recent possibility, suggested in [14] for quantum fields, is the *many-fingered time* (MFT) formulation of Bohmian mechanics, based on the MFT formulation of orthodox many-particle QM [15] and QFT [15, 16]. The purpose of the present paper is to further develop the idea of the MFT Bohmian interpretation introduced in [14]. More specifically, the aim is (i) to present the MFT formulation of Bohmian mechanics for many-particle QM (which was not presented in [14]) and (ii) to improve and correct some of the results and statements on the MFT Bohmian mechanics of fields presented in [14]. The present paper can also be viewed as

complementary to [14], in the sense that the present paper, unlike [14], does not insist on the manifestly relativistic-covariant formulation, but instead discusses the conceptual meaning of the MFT-nature of Bohmian hidden-variable beables more carefully.

Sec. 2 contains a review of the orthodox MFT formulation of many-particle QM, while the corresponding MFT Bohmian interpretation is discussed in Sec. 3. The generalization to QFT is briefly discussed in Sec. 4, after which the conclusions are drawn in Sec. 5.

Throughout the paper, we use units in which $\hbar = 1$.

2 MFT formulation of many-particle QM

A natural starting point towards a relativistic-covariant formulation of many-particle QM is to introduce a kinematical framework in which time is treated on an equal footing with space. Thus, instead of a single-time n -particle wave function $\psi(\mathbf{x}_1, \dots, \mathbf{x}_n, t)$, one introduces a MFT n -particle wave function [15]

$$\Psi(\mathbf{x}_1, \dots, \mathbf{x}_n, t_1, \dots, t_n). \quad (1)$$

However, a MFT formulation can also be introduced independently of the principle of relativity, so in this section, for simplicity, we actually study the nonrelativistic version of the MFT formulation of QM. One of the main purposes of this study is to demonstrate that, with the MFT formulation of QM, the wave-function “collapse” induced by a measurement does not require a preferred notion of simultaneity.

The quantity

$$\rho(\mathbf{x}_1, \dots, \mathbf{x}_n, t_1, \dots, t_n) = |\Psi(\mathbf{x}_1, \dots, \mathbf{x}_n, t_1, \dots, t_n)|^2 \quad (2)$$

is the probability density for finding one particle at the position \mathbf{x}_1 at the time t_1 , another particle at the position \mathbf{x}_2 at the time t_2 , etc. (For a recent generalization of this to the relativistic case, see [17].) When different particles do not interact with each other, then the MFT wave function satisfies n independent local Schrödinger equations

$$\hat{H}_i \Psi = i \frac{\partial}{\partial t_i} \Psi, \quad (3)$$

where

$$\hat{H}_i = -\frac{\nabla_i^2}{2m_i} + V_i(\mathbf{x}_i, t_i), \quad (4)$$

and $i = 1, \dots, n$. It is convenient to introduce a simpler notation $\mathbf{X} \equiv \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$, $T \equiv \{t_1, \dots, t_n\}$. We also introduce global operators

$$\frac{\partial}{\partial T} = \sum_{j=1}^n \frac{\partial}{\partial t_j}, \quad \hat{H} = \sum_{j=1}^n \hat{H}_j. \quad (5)$$

Thus, by summing up the local Schrödinger equations (3), one obtains a single global Schrödinger equation

$$\hat{H} \Psi = i \frac{\partial}{\partial T} \Psi. \quad (6)$$

The dynamics can be described by a Schrödinger equation of the form of (6) even when different particles do interact with each other.

The MFT Schrödinger equation (6) contains the ordinary single-time Schrödinger equation as a special case in which $t_1 = \dots = t_n \equiv t$. The corresponding wave functions are related as

$$\psi(\mathbf{X}, t) = \Psi(\mathbf{X}, t_1, \dots, t_n)|_{t_1=\dots=t_n=t}. \quad (7)$$

However, the instantaneous synchronization in (7) is not more physical than, for example, a relativistically more appealing retarded light-cone synchronization. Indeed, the question of “true” synchronization in relativistic QM can be viewed as analogous to the question of “true” gauge in electrodynamics. In this analogy, $\Psi(\mathbf{X}, T)$ is a “gauge-independent” quantity, whereas $\psi(\mathbf{X}, t)$ resembles the Coulomb gauge in which the electromagnetic potential propagates instantaneously. (Of course, the analogy with gauge theories should not be taken too literally, but note that a similar analogy with gauge theories has been used in [18] as a response to the criticism in [19].)

A normalized solution $\Psi(\mathbf{X}, T)$ of (6) can be written as a linear combination of other orthonormal solutions as

$$\Psi(\mathbf{X}, T) = \sum_a c_a \Psi_a(\mathbf{X}, T). \quad (8)$$

The base $\{\Psi_a\}$ can be chosen such that each Ψ_a is a local product of the form

$$\Psi_a(\mathbf{X}, T) = \psi_{a1}(\mathbf{x}_1, t_1) \cdots \psi_{an}(\mathbf{x}_n, t_n). \quad (9)$$

Thus, the base wave functions $\Psi_a(\mathbf{X}, T)$ do not exhibit a nonlocal entanglement, but a general superposition (8) does.

Now assume that $\psi_{a1}(\mathbf{x}_1, t_1)$ are the eigenstates of some local Hermitian operator that is measured. Such a local measurement induces a *nonlocal* wave-function “collapse”

$$\Psi(\mathbf{X}, T) \rightarrow \Psi_a(\mathbf{X}, T). \quad (10)$$

Now the crucial point is the following: If the local measurement is performed at some particular value of the time t_1 , then it does *not* mean that the *whole* wave function $\Psi(\mathbf{X}, T)$ collapses at the same particular value of time. Namely, fixing the value of t_1 in the collapsed wave function $\Psi_a(\mathbf{X}, T)$ in (10) does *not* fix the values of t_2, \dots, t_n . In this sense, *in the MFT formulation of QM, the wave-function “collapse” does not require any preferred notion of simultaneity*. Thus the MFT formulation of QM can be used to enlighten the Einstein-Podolsky-Rosen effect (see e.g. [20]) and the delayed-choice experiment (we are not aware of any particular reference that explicitly uses the MFT formulation to discuss the delayed-choice experiment).

Concerning the problem of measurement, the only true problem in orthodox QM is to understand a physical mechanism that induces the wave-function “collapse” (10). Such a mechanism is provided by the MFT Bohmian hidden-variable interpretation studied in the next section.

3 MFT Bohmian interpretation of many-particle QM

By writing $\Psi = Re^{iS}$, where R and S are real functions, the complex equation (6) is equivalent to a set of two real equations

$$\sum_{i=1}^n \left[\frac{(\nabla_i S)^2}{2m_i} + V_i(\mathbf{x}_i, t_i) \right] + Q(\mathbf{X}, T) + \frac{\partial S}{\partial T} = 0, \quad (11)$$

$$\frac{\partial \rho}{\partial T} + \sum_{i=1}^n \nabla_i \left(\rho \frac{\nabla_i S}{m_i} \right) = 0, \quad (12)$$

where $\rho = R^2$ and

$$Q = - \sum_{i=1}^n \frac{1}{2m_i} \frac{\nabla_i^2 R}{R}. \quad (13)$$

The conservation equation (12) confirms that it is consistent to interpret $\rho(\mathbf{X}, T)$ as the probability density.

In analogy with the ordinary single-time Bohmian interpretation, we introduce a MFT beable $\mathbf{x}_i(T)$ that satisfies the MFT Bohmian equation of motion

$$\frac{\partial \mathbf{x}_i(T)}{\partial T} = \frac{\nabla_i S}{m_i}. \quad (14)$$

From (14) and (11), one can also derive the MFT quantum Newton equation

$$m_i \frac{d^2 \mathbf{x}_i(T)}{dT^2} = -\nabla_i [V_i(\mathbf{x}_i, t_i) + Q(\mathbf{X}, T)]. \quad (15)$$

In contrast with the ordinary Bohmian interpretation, the beable $\mathbf{x}_i(T) \equiv \mathbf{x}_i(t_1, \dots, t_n)$ *cannot* be interpreted as a trajectory in spacetime. Nevertheless, for $t_1 = \dots = t_n = t$, the beable $\mathbf{x}_i(T)$ reduces to the ordinary Bohmian beable $\mathbf{x}_i(t)$, which, indeed, can be interpreted as a trajectory in spacetime. However, the fundamental ontology is not represented by the synchronization-dependent function $\mathbf{x}_i(t)$, but rather by the synchronization-independent function $\mathbf{x}_i(T)$. (Recall the analogy with gauge theories, discussed in the preceding section.) For any set $T = \{t_1, \dots, t_n\}$, the functions $\mathbf{x}_i(T)$, $i = 1, \dots, n$, uniquely specify the particle positions \mathbf{x}_i . Analogously to the ordinary Bohmian interpretation, Eqs. (14) and (12) imply that the MFT Bohmian interpretation predicts the same probabilities for finding the first particle at the position \mathbf{x}_1 at the time t_1 , the second particle at the position \mathbf{x}_2 at the time t_2 , etc., as does the orthodox interpretation of MFT QM. Moreover, if the wave functions $\Psi_a(\mathbf{X}, T)$ in (9) do not overlap in at least a part of the configuration space, so that $\Psi_a(\mathbf{X}, T)\Psi_{a'}(\mathbf{X}, T) = 0$ for $a \neq a'$, then some of the degrees of freedom can be interpreted as the degrees of freedom of the measuring apparatus. Consequently, analogously to the ordinary Bohmian interpretation, the MFT Bohmian interpretation predicts the same probabilities (equal to $|c_a|^2$) for the effective “collapse” (10) as does the orthodox MFT interpretation. In the MFT Bohmian interpretation, the effective “collapse” occurs because the beables $\mathbf{x}_i(T)$ take values from the support of one and only one of the nonoverlapping wave functions $\Psi_a(\mathbf{X}, T)$.

Is the ontology represented by $\mathbf{x}_i(T)$ in contradiction with the fact that, for example, we can observe the particle position \mathbf{x}_1 at the time t_1 *without* measuring the times t_2, \dots, t_n ? Although there is no beable corresponding to the quantity \mathbf{x}_1 at time t_1 , the beables $\mathbf{x}_i(T)$ determine the wave function Ψ_a to which Ψ will effectively “collapse”. If the functions Ψ_a in (9) are such that $\psi_{a1}(\mathbf{x}_1, t_1)$ are eigenfunctions of the local position operator \mathbf{x}_1 , then such a collapse can be viewed as a measurement of $\mathbf{x}_1(t_1)$, despite the fact that there is no beable corresponding to $\mathbf{x}_1(t_1)$. Indeed, this is just an example of a measurement of an *unpreferred* observable in the Bohmian interpretation, such as momentum or energy in the ordinary single-time Bohmian interpretation. In the MFT Bohmian interpretation, the preferred observables are $\mathbf{x}_i(T)$, but the general theory of quantum measurements explains measurements of all other observables, with the same statistical predictions as in the orthodox interpretation.

Let us also compare the nonlocality features in the ordinary and MFT Bohmian interpretations. In the ordinary single-time Bohmian interpretation, the ontology of hidden variables is classical at the kinematical level (given by local particle trajectories), whereas the quantum nonlocality is realized only on the dynamical level (encoded in the instantaneous nonlocal quantum potential). In contrast, in the MFT Bohmian interpretation, the ontology is nonclassical and nonlocal already at the kinematical level, because, in $\mathbf{x}_i(T)$, \mathbf{x}_i is a function not only of t_i , but of *all* t_1, \dots, t_n . One may complain that the function $\mathbf{x}_i(T)$ is difficult to visualize, but one should not be worried about that, given the fact that it is certainly not more difficult to visualize than the MFT wave function $\Psi(\mathbf{X}, T)$. One should recall that, historically, the aim of the Bohmian interpretation was *not* to restore the classical ontology in QM (although, perhaps surprisingly,

the ordinary Bohmian interpretation has done that), but rather to find *some* nonlocal beable that could reproduce the predictions of orthodox QM.

We also note that the MFT formalism enables one to formulate the Bohmian interpretation of many-particle systems in an explicitly relativistic-covariant way. This will be the subject of a separate paper, but we anticipate that it can be done by combining the results of the present paper with those of [17].

4 MFT Bohmian interpretation of QFT

The purpose of the present section is to generalize the results of the preceding sections to the case of QFT. However, as the MFT Bohmian interpretation of QFT has already been discussed in detail in [14], in this section we only briefly outline the main points of the generalization, emphasizing those aspects that have been treated incorrectly in [14], or have not been discussed at all.

Instead with a discrete set $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$, field theory deals with a continuous set of values of fields at different points, $\phi = \{\phi(\mathbf{x})\}$, at all space points \mathbf{x} . Similarly, the discrete set of times $T = \{t_1, \dots, t_n\}$ is replaced with a continuous set $T = \{T(\mathbf{x})\}$. The quantum state is represented by a wave functional $\Psi[\phi, T]$. The QFT analog of (3) is known as the Tomonaga-Schwinger equation [15, 16]. Introducing the operator

$$\frac{\partial}{\partial T} = \int d^3x' \frac{\delta}{\delta T(\mathbf{x}')}, \quad (16)$$

the QFT analog of the MFT Bohmian equation of motion (14) is

$$\frac{\partial \phi(\mathbf{x}; T]}{\partial T} = \frac{\delta S}{\delta \phi(\mathbf{x})}. \quad (17)$$

(On the right-hand side, it is understood that $\phi(\mathbf{x}')$ is replaced with $\phi(\mathbf{x}'; T]$ at all points \mathbf{x}' .) However, in [14] it was stated that the fundamental MFT Bohmian equation was not the global MFT equation (17), but a local MFT equation

$$\frac{\delta \phi(\mathbf{x}; T]}{\delta T(\mathbf{x}')} = \delta^3(\mathbf{x} - \mathbf{x}') \frac{\delta S}{\delta \phi(\mathbf{x})}. \quad (18)$$

Indeed, if (18) is satisfied, then (18) implies (17). However, although Eq. (17) is consistent, Eq. (18), in general, may not be consistent. In general, the right-hand side of (18) depends not only on $T(\mathbf{x})$, but on the *whole* function T at all points \mathbf{x}' . On the other hand, the δ -function on the right-hand side of (18) implies that $\phi(\mathbf{x}; T]$ on the left-hand side does not depend on the whole function T , but only on $T(\mathbf{x})$. However, for $\mathbf{x}' = \mathbf{x}$, this implies that the left-hand side of (18) depends only on $T(\mathbf{x})$, whereas the right-hand side depends on the whole function T , which is inconsistent. Thus, the correct MFT Bohmian equation of motion is (17), rather than (18). Consequently, contrary to the claim in [14], the MFT Bohmian beable is, in general, a genuine MFT field $\phi(\mathbf{x}; T]$, rather than a local field $\phi(\mathbf{x}, T(\mathbf{x}))$. Nevertheless, the local appearance of fields can be explained by the theory of quantum measurements, analogous to that in the preceding section.

It is also interesting to study the conditions under which the local MFT Bohmian equation of motion (18) could still be consistent. One such condition is a wave functional that has a form of a local product analogous to (9), but such a condition is not sufficiently general. A more general condition is *any* quantum field theory that contains *gravity* as one of the quantized fields.

Namely, the theories that contain gravity have a property of diffeomorphism invariance, which implies that the Hamiltonian always vanishes on-shell. Consequently, instead of a functional Schrödinger or Tomonaga-Schwinger equation, one deals with the Wheeler-DeWitt equation [21, 22, 23, 24, 25]

$$\hat{\mathcal{H}}(\mathbf{x})\Psi[g, \phi] = 0, \quad (19)$$

where $\hat{\mathcal{H}}(\mathbf{x})$ is the Hamiltonian-density operator, g represents the 3-metric and ϕ represents all other “matter” fields. Since the wave functional $\Psi[g, \phi]$ does not depend on time (either on t or on T), it is consistent to postulate a local MFT Bohmian equation of motion of the form of (18) for both ϕ and g .

Finally, let us note that it is straightforward to write all equations of this section in a manifestly general-covariant form, by using the formalism presented in [14]. In particular, this leads to a covariant version of the Bohmian interpretation of quantum gravity, which represents an improvement of the noncovariant Bohmian interpretation of quantum gravity studied in [6, 26, 27, 28, 29].

5 Conclusion

The MFT formulation of QM and QFT allows a formulation of quantum theory that does not require a preferred definition of simultaneity, which alleviates the problem of relativistic-covariant formulation of quantum theory, including the problem of simultaneity of the wave-function “collapse”. The corresponding Bohmian interpretation leads to new MFT beables that also do not require a preferred definition of simultaneity. These MFT beables have a manifest nonlocal nature already at the kinematical level. Nevertheless, the observed local appearance of particles and fields can be recovered by studying the theory of quantum measurements.

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References

- [1] R. Medina, Annales de la Fondation Louis de Broglie 24 (1999) 129, quant-ph/0508014.
- [2] J.S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, 1987.
- [3] D. Bohm, Phys. Rev. 85 (1952) 166, 180.
- [4] D. Bohm, B.J. Hiley, Phys. Rep. 144 (1987) 323.
- [5] D. Bohm, B.J. Hiley, P.N. Kaloyerou, Phys. Rep. 144 (1987) 349.
- [6] P.R. Holland, Phys. Rep. 224 (1993) 95.
- [7] P.R. Holland, The Quantum Theory of Motion, Cambridge University Press, Cambridge, 1993.
- [8] H. Nikolić, Found. Phys. Lett. 17 (2004) 363; H. Nikolić, Found. Phys. Lett. 18 (2005) 123.
- [9] D. Dürr, S. Goldstein, K. Münch-Berndl, N. Zanghì, Phys. Rev. A 60 (1999) 2729.

- [10] G. Horton, C. Dewdney, J. Phys. A 37 (2004) 11935.
- [11] H. Nikolić, Eur. Phys. J. C 42 (2005) 365; H. Nikolić, hep-th/0512186; H. Nikolić, hep-th/0601027.
- [12] K. Berndl, D. Dürr, S. Goldstein, N. Zanghì, Phys. Rev. A 53 (1996) 2062.
- [13] H. Nikolić, Found. Phys. Lett. 18 (2005) 549; H. Nikolić, quant-ph/0512065.
- [14] H. Nikolić, Phys. Lett. A 348 (2006) 166.
- [15] S. Tomonaga, Prog. Theor. Phys. 1 (1946) 27.
- [16] J. Schwinger, Phys. Rev. 74 (1948) 1439.
- [17] H. Nikolić, quant-ph/0602024.
- [18] A. Peres, Phys. Rev. A 64 (2001) 066102.
- [19] H. Nikolić, Phys. Rev. A 64 (2001) 066101.
- [20] P. Ghose, D. Home, Phys. Rev. A 43 (1991) 6382.
- [21] J.A. Wheeler, in Battelle Rencontres, eds. C.M. DeWitt, J.A. Wheeler, Benjamin, New York, 1968.
- [22] B. DeWitt, Phys. Rev. 160 (1967) 1195.
- [23] T. Padmanabhan, Int. J. Mod. Phys. A4 (1989) 4735.
- [24] K. Kuchař, in Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics, World Scientific, Singapore, 1992.
- [25] C.J. Isham, gr-qc/9210011.
- [26] S. Goldstein, S. Teufel, quant-ph/9902018.
- [27] N. Pinto-Neto, E.S. Santini, Phys. Rev. D 59 (1999) 123517.
- [28] N. Pinto-Neto, E.S. Santini, Gen. Rel. Grav. 34 (2002) 505.
- [29] A. Shojai, F. Shojai, Class. Quant. Grav. 21 (2004) 1.